A Framework for Quality of Service in a Multiple Access Network

Easwar Vivek M, Venkatesh Ramaiyan
Department of Electrical Engineering
Indian Institute of Technology Madras
Chennai 600036, India
Email: {ee10s045, rvenkat}@ee.iitm.ac.in

***Abstract*—In this work, we study a problem of supporting real time traffic with hard delay constraints in an unreliable wireless data communication channel. A fixed number of nodes N share a common wireless channel using a centralized scheduler at the access point or the base station. We assume that the wireless channel is slotted and the nodes are synchronized. We assume that the data communication channel is unreliable, where a communication can fail in a slot with a known probability. Also, we assume that all packet communications fail in the presence of simultaneous transmissions. Further, we assume that the data packets have hard delay constraints. In this scenario, we are interested in the long time average rates achievable for the real time traffic in the wireless network. In [1], the authors have characterized the rate region of such a network by identifying the feasible rate vectors. In this work, we provide a characterization based on the average rate vector achievable in a slot. We, then, extend the characterization to study the wireless network in a multiple access scenario. We show that the multiple access scenario improves the rate region of the wireless network and we propose throughput optimal and network utility maximizing schedulers for the multiple access wireless network. Finally, we report performance analysis of the proposed scheduling strategy and discuss its advantages.**

I. Introduction

Provisioning quality of service for real time traffic in a wireless network is crucial and is gaining importance with applications of voice and video streaming. They are also critical for sensor network applications including real time surveillance and applications that involve network control. A common and critical feature in enabling such applications is the emphasis on delay encountered by the packets in the wireless network. While best effort traffic has minimal delay constraints, real time traffic can have strict constraints on delay including hard delay constraints, probabilistic and average delay constraints. In this paper, we consider an unreliable, time varying wireless network and we study the problem of provisioning resources for a real time traffic with hard delay constraints. We consider a framework similar to that reported in [1] to support the QoS and extend it for a multiple access network scenario.

/A paragraph on the paper theory of QoS/. In [1], the authors have proposed a framework to deal with delay, throughput and channel reliabilities. They have obtained the necessary conditions such that the requirements of a set of users can be met. These necessary conditions are the conditions to be met by the workloads brought in by the users. They further went on to prove that these conditions are indeed sufficient. They verify these necessary and sufficient conditions for all the subsets of users to admit a new user into the sytem . Their admission control algorithm verifies if the new set of users is feasible i.e., if the requirement rate vector is feasible.

We use a similar framework but characterise the system by obtaining the expected rate vector achievable in each period (to be defined later). We identify few schedules and evaluate the average throughput obtained using these schedules. We verify that these schedules indeed form the boundary of all the rate vectors achievable in a period. We therefore obtain the set of all the rate vectors achievable in a period. This knowledge of the possible rate vectors will be helpful in designing scheduling policies to meet the requirements of the users.

We then propose to extend the framework in [1] to a multi access scenario. The rate region obtained by this extended framework is larger than the previous one implying better performance interms of throughput. The access point instead of polling the users does a RTS/CTS exchange with the users at the beginning of everyslot to schedule the users. We obtain the rate region for this extended framework by obtaining the expected value of achievable rate vector in each period. We also obtain few simpler conditions under which the rate region is improved. We then propose throughput and utility optimal schedulers that are feasibility optimal. We also analyse the performance of these scheduling algorithms that operate in this framework.

A. Literature Survey

Characterising the stability region/rate region of systems and studying throughput optimal schedulers and their delay performance has been an area of great interest. In [2] the optimal order of delay for a certain arrival process has been studied. The problem of delay constraints in distributed networks has been studied in [3]. The problem of scheduling real time traffic with packets having heterogenous deadlines has been studied in [4]. utility maximisation problem with delay constraints has been studied in [5]. Finding the rate region of the system and designing an online scheduler to achieve the same in a partial channel state information scenario has been dealt with in [6]. Supporting both elastic and inelastic trafic has been discussed in []

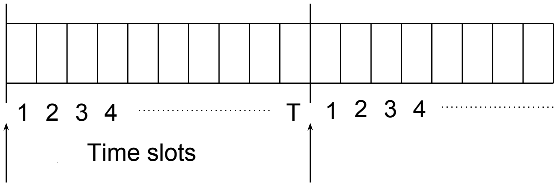

Fig. 1. Structure of the frame

II. Network model

We consider an infrastructure wireless network with a base station or an access point and a fixed number N of wireless users. The wireless channel is slotted and the nodes are synchronized. Further, we assume a notion of a frame, where a frame is composed of τ slots. We assume that the wireless nodes are frame synchronized as well. Packets are generated at the beginning of every frame, one per node per frame (an uplink traffic model). The packets have strict delay constraints and the packets need to be communicated to the access point within the frame, i.e., the packets have a strict delay constraint of τ slots.

We consider an unreliable ON/OFF $(0/1)$ wireless channel, where a packet communication between the access point and the user can fail in a slot with a Bernoulli probability. We assume that the channel of user i is ON (1) with probability p_i in any slot and the event is independent across time and across users. We consider a centralized scheduling strategy where the access point coordinates transmissions based on the QoS requirements of the nodes. For example, the access point can schedule node i in a slot t. The communication is enabled by a control packet transmission by the access point. The control packet is successfully received by the node i if the channel is ON (with probability p_i). Upon successful reception of the control packet, the node i responds to the access point with the data packet. We assume that the channel condition is constant within a frame, and hence, one data packet can be successfully transmitted upon reception of a control packet. We assume that the slot duration is such that it permits a control packet and a data packet (and a possible acknowledgement) in every slot. The framework can also be used to study downlink transmissions using a RTS/CTS control packet exchange.

In [1], the authors have studied a similar framework where the access point polls a single user in every slot to schedule the user. In this work, we extend the framework, where we permit multiple access in a slot by polling a subset of the users to transmit in every slot. The access point broadcasts a control packet destined to a subset of users in every slot. All the intended nodes that successfully decode the packet (nodes that were addressed and whose channel condition is ON in the slot) respond with a data packet. We assume that the communication is successful only if a lone user responds to the control packets and all packet transmissions are lost in the presence of simultaneous communication. In this setup, we study the problem of identifying an optimal subset of users in every slot to schedule.

The performance metric of interest in this work is the long time average throughput vector. Let $D_i(t)$ be the random variable that represents the number of packets transmitted in a slot. The long time average throughput of user i is then defined as

$$d_i = \liminf_{t\to\infty} \frac{1}{t}\sum_{k=1}^{t} D_i(k)$$

Of course, we require that $D_i(t) \in \{0,1\}$ in every slot t, $\sum_{i=1}^{N} D_i(t) \leq 1$ for all t and $\sum_{k=nT+1}^{nT+t} D_i(k) = 1$ in every frame. Let $(d_1, d_2, \cdots, d_N)$ be a feasible long time average rate vector, i.e., there exists some scheduling strategy, possibly non-causal, that can achieve this long time average rate vector. Then, the rate region of the wireless network, $\mathcal{C}$ is the set of all feasible rate vectors $(d_1, d_2, \cdots, d_N)$. The network objective considered in this work is to identify a scheduler that can support any feasible rate vector for the wireless network and to maximize a network utility on the rate region for a given network objective.

III. Rate Region of the Wireless Network

In [1], the authors have studied the problem of scheduling users with hard delay requirements when a single user is polled in a slot. They characterize the rate region of the wireless network using the load constraints on the network. In Theorem ref, the authors show that an N-tuple $(d_1, d_2, \cdots, d_N)$ is a feasible rate vector if

$$\sum_{i\in\mathcal{S}} \frac{d_i}{p_i T} \leq 1 - \mathcal{E}[I_{\mathcal{S}}]$$

where $\mathcal{S}$ is any subset of $1, 2, \cdots, N$ and $\mathcal{E}[I_{\mathcal{S}}]$ is the expected number of idle slots in a frame when the set of $\mathcal{S}$ users are scheduled.

In this work, we like to characterize the rate region directly, by characterizing the average rate vector feasible in every frame.

The capacity of a wireless channel has been studied earlier in a number of works for a variety of network scenarios (see [7], [2], [6]). We note that the capacity of the delay constrained wireless network can be similarly defined by identifying that the channel in every frame is i.i.d. and the average rate vector in every frame can be computed by identifying the set of schedules and by computing the rate vector associated with the schedule.

As the ON probabilities of the channels are identical across slots and frames, the expected throughput obtained for any frame would be same as that of the system.

A simplistic view of schedule would be the sequence in which packets are scheduled in a frame, and $(N+1)!$ such permutations are feasible for a given network. We expect that the other combinations of schedules are irrelevant and need not be studied. Each such permutation would result in a particular expected rate vector in a frame and hence a particular point on the rate region. The number of such points can be obtained

by considering the possible permutations involving each set of users.

Let S be the set of schedules feasible for a given channel.

Any number of users can be scheduled in a frame and in any order. Each order of any K $(K \in 1, 2, 3 \cdots N)$ is a schedule and would achieve an expected rate vector for each frame. All the permutations of all the subsets of the users would form the possible set of schedules in a slot. If S' is the schedule in any frame , then S' $\in S_k$ for any k. $S_k \in S : k \in 1, 2, \cdots N$ and S_k is the set of all permuatations of the k selected users.

In this work, the expected rate vector obtained in a frame is an equivalent to the expected rate vector of a slot defined in the works [8]. Hence it can be inferred that the convex hull of all such points shall be the rate region of the system.

Assume N users $(1, 2, 3...N)$ in the system. The number of permutations involving the scheduling of user 1 and no other user after him is ${}^{N-1}P_0$. Similarly the number of permutations with only r users among the other users being scheduled after user 1 is ${}^{N-1}P_r$. These ${}^{N-1}P_r$ ways are the different schedules with different priority orders among r users. Hence the total number of schedules with user 1 being given priority over any other user is

$$\sum_{r=0}^{N-1} {}^{N-1}P_r \tag{1}$$

So the total number of ways in which N users can be scheduled with different priority orders and different number of users is

$$N * \sum_{r=0}^{N-1} {}^{N-1}P_r \tag{2}$$

These are the different corner points of the rate region which is now a N dimensional polymatroid. [9]. (2) is the number of corner points excluding the origin. The minimalistic set of points, with the convex combination of which we can obtain the whole rate region can be obtained by considering the schedule points where all the N users are scheduled. These will be $N!$ points. These $N!$ points represent the dominant face of the rate region.

Each of the necessary conditions given corresponds to a face on the rate region. Hence there are $2^N - 1$ faces on the rate region which is the total number of subsets of N users. Any face obtained by a necessary condition has all the points, obtained by schedules which involve all the users in the subset used for the necessary condition, as corner points.

A. Rate Region for a 2 User Network

We will now illustrate that the rate region described by the two approaches are identical by illustrating for a 2 user wireless network.

The necessary and sufficient conditions involving two users as proved in [1] would be

$$\frac{d_1}{p_1 T} + \frac{d_2}{p_2 T} \leq 1 - \mathcal{E}[I_{\mathcal{S}}]$$

$$\frac{d_1}{p_1 T} \leq 1 - \mathcal{E}[I_1]$$

$$\frac{d_2}{p_2 T} \leq 1 - \mathcal{E}[I_2]$$

These three inequalities represent the rate region of the two user system. These expressions considered with equality shall represent the boundary of the rate region.

To obtain the rate region in the proposed approach, consider the schedule in which user 1 is scheduled in all the time slots in a frame. In each frame, he obtains an expected throughput of

$$1 - (1 - p_1)^T \tag{3}$$

This results in a point on the user 1 axis of the rate region which is

$$\Big((1 - (1 - p_1)^T, 0\Big) \tag{4}$$

Now, if user 2 is scheduled as soon as user 1 is successful in every frame, he would obtain a throughput of

$$\sum_{k=1}^{T} (1 - p_1)^{k-1} p_1 (1 - (1 - p_2)^{T-k}) \tag{5}$$

This would result in a point directly above the point obtained previously. Thus the point obtained by prioritising user 1 over user 2 in every time frame is

$$\left(1 - (1 - p_1)^T, \sum_{k=1}^{T} (1 - p_1)^{k-1} p_1 (1 - (1 - p_2)^{T-k})\right) \tag{6}$$

Similarly a point is obtained on user 2 axis by allocating all the slots to him. Also, the point obtained by prioritising user 2 over user 1 in every time frame is

$$\left(\sum_{k=1}^{T} (1 - p_2)^{k-1} p_2 (1 - (1 - p_1)^{T-k}), 1 - (1 - p_2)^T\right) \tag{7}$$

The convex hull of these points gives the Rate Region[7].

These four achievable corner points satisfy the necessary condition of equation (1) with equality, thus proving that the region obtained is indeed the Rate Region. The proof of this can be found in Appendix A.

IV. Extended Network Model

Now we extend the Network Model to include an RTS and a CTS at the beginning of every slot. Each slot is assumed to be long enough to accomodate a RTS, CTS and the time taken for one complete packet transmission along with the time taken for the acknowledgement to reach the destination. At the beginning of each slot, the base station broadcasts a RTS to the users. The users with an ON channel respond with a CTS [10]. If the base station observes a single success, the user which responded is scheduled in the time slot. This utilises the opportunism among the users[11], [12]. If it encounters

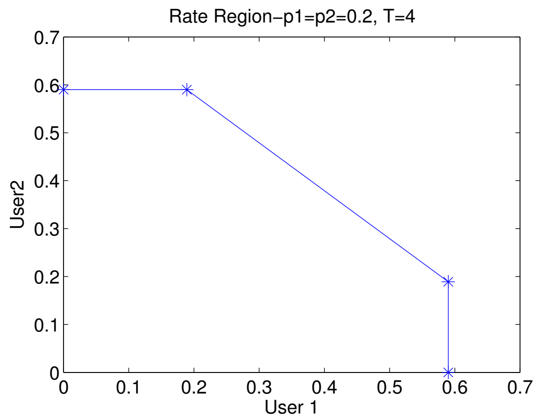

Fig. 2. Rate Region with two users and $p_1 = p_2 = 0.2, T = 4$

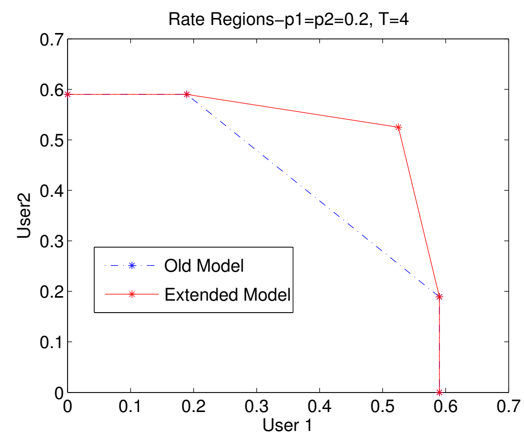

Fig. 3. Comparision of Rate Regions $p_1 = p_2 = 0.2, T = 4$

a collision during the CTS phase, the slot is wasted. Instead of braodcasting to all the users, the base station can choose a subset of users [6] and multicast the RTS to them to minimise the probability of collision in the slot.Similar to the previous model, a Rate Region can be obtained for this system by considering all the schedules. Along with the points obtained in the previous model, the rate region has all the points obtained by using different subsets for each slot. Thus the number of points that are added to the rate region are the total number of subsets of N users i.e., $2^N - 1$. Hence the total number of schedule(corner) points in the rate region obtained by the extended model is the summation of the number in (2) and $2^N - 1$. Thus the rate region would be a convex hull of all the points obtained in the previous model and the points obtained by polling different subsets of users in each slot.

Rate Region

A two user system is considered for simplicity. Consider the schedule in which only user 1 is requested for data by sending RTS in every slot. The expected throughput obtained is similar to the point (4) obtained by the previous model. Also, a point similar to (6) is obtained by sending RTS to user 2 as soon as user 1 is successful. Two such points are obtained with user 2 being given priority over user 1.

Now, consider the schedule in which both the users are sent the RTS at the beginning of every slot. If channel of only one user is ON, his CTS will be successfully received at the base station and he will be scheduled in the slot. The slot is wasted when both the users have an ON channel and their CTSs collide. If both the users see a OFF channel, the slot is wasted by default. This schedule also results in a point on the rate region.

The user 1 throughput with the schedule is

$$\begin{aligned}\sum_{k=1}^{T}\Big[&((1-p_2)(1-p_1))^{k-1}(1-p_2)p_1+\\ &\sum_{l=0}^{k-2}((1-p_2)(1-p_1))^{l}p_1(1-p_1)^{k-l-2}\Big]\end{aligned} \tag{8}$$

The user 2 throughput is

$$\begin{aligned}\sum_{k=1}^{T}\Big[&((1-p_1)(1-p_2))^{k-1}(1-p_1)p_2+\\ &\sum_{l=0}^{k-2}((1-p_1)(1-p_2))^{l}p_1(1-p_2)^{k-l-2}\Big]\end{aligned} \tag{9}$$

The convex hull of these points gives the rate region.

The region for the extended network model with two users has been shown in figure 2.

V. Comparision of Rate Regions

As the Extended model can execute all the schedules that are possible in the previous model, its rate region is atleast as large as that of the one obtained for it. In the new model, instead of polling users one after the other and transmitting till a particular user is successful, users are chosen depending on whose channel is ON.Thus, the new model utilises opportunism among the users to reduce slot wastage due to failed transmissions. Thus the rate region is larger than the previous model. This expansion of rate region due to the exploitation of opportunism is more pronounced especially when the probabilities of success of the users is low. A comparision of the rate regions has been shown in fig 3

Conditions for Improvement

The rate region is expanded only when the set of probabilities of channels being ON meet a certain criterion. This is because, if the ON probabilities are very high, more slots are lost because of collisions during the CTS phase.

Consider a system when $T = 1$. The rate region is expanded if the total success probability of the system in a slot because of the new model is atleast as large as the success probability of any of the user in the system.

The probability of success in a slot in two user system when $T = 1$ is

$$p_1(1-p_2) + p_2(1-p_1) \tag{10}$$

A sub-optimal condition for which the rate region is expanded is

$$p_1(1-p_2)+p_2(1-p_1) \geq p_1 \tag{11}$$

$$p_1(1-p_2)+p_2(1-p_1) \geq p_2 \tag{12}$$

Sovling these equations results in

$$p_1 \leq \frac{1}{2}; p_2 \leq \frac{1}{2} \tag{13}$$

Thus if the ON probabilities of the users are less than $\frac{1}{2}$ the rate region is definitely expanded for two user system.

Theorem 1. *For a N user system, the rate region is expanded if $p_i \leq \frac{1}{N}$ for $T = 1$*

Proof:

The rate region of a two user system that does not use opportunism and polls any user in a slot, when $T = 1$ is the convex hull of the points $(p_1, 0), (0, p_2)$. These points for a 3 user system will be $(p_1, 0, 0), (0, p_2, 0), (0, 0, p_3)$. The rate region of the system is the region of the first quadrant lying towards the origin of the plane described by the three points.

Thus the rate region is

$$\frac{x_1}{p_1}+\frac{x_2}{p_2}+\frac{x_3}{p_3} \leq 1 \tag{14}$$

where (x_1, x_2, x_3) is the point

The equation of the plane for N users will be

$$\sum_{i=1}^{N} \frac{x_i}{p_i} \leq 1 \tag{15}$$

If it can be shown that when each $p_i \leq 1$ the point obtained by using opportunism among N lies beyond the equation of plane in 15, the proof shall be completed.

The point obtained by using opportunism among N users is

$$\big(p_1 \prod_{i \neq 1}(1-p_i), p_2 \prod_{i \neq 2}(1-p_i), ..., p_N \prod_{i \neq N}(1-p_i)\big) \tag{16}$$

Substituting 16 in 15 gives

$$\sum_{i=1}^{N} p_i \prod_{j \neq i}(1-p_j) \tag{17}$$

If $p_i \leq \frac{1}{N} \forall i$, then 17 is always greater than or equal to $(1-\frac{1}{N})^{N-1}$ which is inturn greater than or equal to 1 $\forall N \geq 2$

Hence the point obatined by the scheduling N users opportunistically lies beyond the plane in 15

Hence the theorem.

■

Theorem 2. *Expansion of rate region for $T = 1$ implies expansion for $T \geq 1$*

Proof:

■

For a N user system, the rate region is trivially expanded if any two users satisfy the condition given in (13)

VI. Throughput Optimal and Utility Optimal Schedulers

Using the rate region, schedulers that can achieve every point on the rate region i.e., schedulers that are throughput optimal can be obtained. A straightforward scheduler can be the the scheduler that expresses every point on the rate region as a convex combination of the obtained corner points and does the corresponding time sharing between the corner point schedules. But this scheduler does not take the deficit that needs to be served for each user in each slot.

Scheduler that uses deficit

The users come with certain throughput(or delivery ratio) requirements, if the requirement rate vector is within the rate region, those users are served. The difference between the throughput required and throughput obtained by any user is the *deficit* of the user.

Let $d_i(n)$ be the deficit of user i in slot n. This deficit can be looked at as the queue length for the user i. Then in every slot the scheduler chooses a subset S_k of S (the set of all users)

$$\arg\max_{S_k \in S} \sum_{i \in S_k} (d_i p_i \prod_{j \neq i} p_j) \tag{18}$$

If a particular user is already successful in the period, the subsets of other users excluding the users are looked at. The users of the chosen subset S_k are sent a request via RTS mulitcast. These users respond with a CTS each. If only one CTS is received successfully the user from which it is received is scheduled.

Here, $p_i \prod_{j \neq i} p_j$ is the expected throughput obtained by user i in the slot. Hence the scheduler is a

$$\arg\max q_i E(r_i) \tag{19}$$

schdeuler which is throughput optimal []

Two user illustration

A two user system with similar throughput requirements and at a similar distance, here 3 units is considered. The users are assumed to encounter rayleigh fading and the probability that the channel is ON is the non-outage probability. Frame length of 7 is considered and the extended network model has been simulated. The obtained throughputs are plotted in the fig 4

VII. Performance Analysis

The performance of the system that uses the Extended model can be compared to the previous model. A cellular network with N users and a cell radius of r is assumed All the users are assumed to undergo rayleigh fading apart from pathloss because of their distance from the base station.

Each user is assumed to have similar throughput requirements

The performance of the system with increasing N and with different radii r for a fixed N have been looked at.

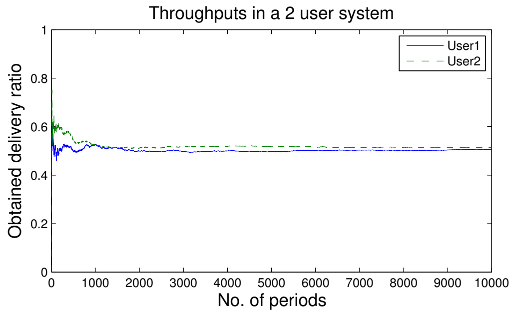

Fig. 4. Throughputs obtained using Extended Network Model

VIII. Conclusion and Future Work

An opportunistic strategy to improve the rate region of an existing model has been proposed. Few simple conditions which if satisfied result in the expansion of the rate region have been found out. Also, it has been shown that all the points of the rate region can infact be obtained by a feasibility-optimal scheduler. This strategy can further be used to provide short-term throughputs to the users along with providing delay guarantees in similar wireless systems.

Appendix

Proof that the corner points staisfy the necessary conditions given in [1]

The given necessary condition for two users is

$$\frac{q_1}{p_1 T} + \frac{q_2}{p_2 T} \leq 1 - I_s \tag{20}$$

I_s corresponds to the expected fraction of idle time in each T

We shall calculate the L.H.S. of the equation and show that it is equal to the R.H.S. when the point 6 is substituted in it.

When the point 6 is substituted in 20,the L.H.S. results in

$$\frac{1-(1-p_1)^T}{p_1 T} + \frac{\sum_{k=1}^{T}(1-p_1)^{k-1}p_1(1-(1-p_2)^T-k)}{p_2 T} \tag{21}$$

which evaluates to

$$\frac{p_1^2(1-(1-p_2)^t + p_2^2(1-p_1)^T - 1)}{p_1 p_1 (p_1 - p_2) T} \tag{22}$$

We now evaluate the R.H.S. of equation (20). In (20) I_s corresponds to the fraction of expected idle time in a frame. The expected idle time per frame can be evaluated as

$$\sum_{i=0}^{T-2} \sum_{k=0}^{T-(i+2)} i(1-p_1)^k p_1 (1-p_2)^{T-(k+i+2)} \tag{23}$$

where the first summation corresponds to the idle time i and the second summation corresponds to the different cases which can result in the same idle time.

Expression (23) evaluates to

$$\frac{p_1^2(1-p_2)^T + p_1^2 p_2 T - p_1^2 - p_2^2(1-p_1)^T - p_1 p_2^2 T + p_2^2}{p_1 p_2 (p_1 - p_2)} \tag{24}$$

The fraction of non idle time in a slot is obtained by $1 - (23)/T$ which evaluates to

$$\frac{p_1^2(1-(1-p_2)^T + p_2^2(1-p_1)^T - 1)}{p_1 p_1 (p_1 - p_2) T} \tag{25}$$

Expressions (22) and (25) are the same showing that the point (6) satisfies the equation of the line corresponding to equation (20) Similarly it can be shown that point (7) also satisfies equation(20)

This implies that the line joining the two achievable points obtained is indeed the necessary condition involving two users making them the sufficient conditions. Hence these points form the boundary of the rate region.

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